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RESEARCH ARTICLE

Ambiguous implementation: the partition model

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Abstract In a partition model, we show that each maximin individually rational and ex ante maximin efficient allocation of a single good economy is implementable as a maximin equilibrium. When there are more than one good, we introduce three conditions. If none of the three conditions is satisfied, then a maximin individually rational and ex ante maximin efficient allocation may not be implementable. However, as long as one of the three conditions is satisfied, each maximin individually rational and ex ante maximin efficient allocation is implementable. Our work generalizes and

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extends the recent paper of de Castro et al. (Games Econ Behav 2015. doi:10.1016/j.geb.2015.10.010).

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1 Introduction

We study the implementation of maximin individually rational and ex ante maximin efficient allocations in an *ambiguous asymmetric information economy*. The implementation notion is the same as in de Castro et al. (2015). That is, we check whether each maximin individually rational and ex ante maximin efficient allocation can be reached through a mechanism as a maximin equilibrium. In a maximin equilibrium, each agent maximizes the payoff that takes into account the worst actions of all the other agents against him and also the worst state that can occur. This paper differs from de Castro et al. (2015) in that we consider a more general setup, which includes de Castro et al. (2015) as a special case. This new and more general framework allows us to consider the standard exchange economies with differential information that have been studied in the literature, e.g., Angelopoulos and Koutsougeras (2015), Castro et al. (2011), Yannelis (1991) among others.

In particular, we adopt a partition model and indicate that the results of this paper cannot be captured by the type model of de Castro et al. (2015). As a matter of fact, the type model cannot capture the standard two persons economies as it is shown in Sect. 3. Our economy consists of a finite set of states of nature, a finite set of agents, each of whom is characterized by an *information partition*, a *multi-prior* set, a *random initial endowment* and an *ex post utility function*. Furthermore, the agents have maximin preferences.

In an ambiguous asymmetric information economy, de Castro and Yannelis (2009) showed that any efficient allocation is incentive compatible with respect to the maximin preferences.[1] This is not true in the standard expected utility (Bayesian) framework. Indeed, an efficient allocation may not be incentive compatible with respect to the Bayesian preferences as it was shown by Holmström and Myerson (1983). Furthermore, Palfrey and Srivastava (1987) showed that under the Bayesian preferences, neither efficient allocations nor core allocations define implementable social choice correspondence, when agents are incompletely informed about the environment. In a recent paper, de Castro et al. (2015) showed that if an ambiguous asymmetric information economy can be represented by the standard type model of the implementation literature, then each maximin individually rational and ex ante maximin efficient allocation is implementable as a maximin equilibrium.

[1] A related work is done by Bosmans and Ooghe (2013). They showed that maximin is the only criterion satisfying anonymity, continuity, weak Pareto and weak Hammond equity.

However, many economies are not representable by the standard type model. Our partition model includes these economies. We illustrate with an example that in such an economy, the agents may strictly prefer a maximin individually rational and ex ante maximin efficient allocation to their initial endowment. Consequently, the problem of implementation in these economies is relevant and interesting.

The main result of the paper is that each maximin individually rational and ex ante maximin efficient allocation of a single good economy is implementable. When there are more than one good, we characterize three sufficient conditions. As long as one of the three conditions is satisfied, each maximin individually rational and ex ante maximin efficient allocation is implementable. We show by means of examples that if none of the three conditions holds, then a maximin individually rational and ex ante maximin efficient allocation may not be implementable. However, the three conditions are not necessary for implementation.

Since the concepts *maximin core allocations*, *maximin value allocations* and *maximin Walrasian expectations equilibrium allocations* defined in de Castro and Yannelis (2009), Angelopoulos and Koutsougeras (2015), He and Yannelis (2015) are all maximin individually rational and ex ante maximin efficient, the implementation of these allocations in a more general model is captured by this paper. We differ from de Castro et al. (2015) in that we allow the players' reports to be incompatible. It follows that the type model of de Castro et al. (2015) can be converted into a partition model. However, the converse is not true in general as we will show in Sect. 3.

The paper is organized as follows. Section 2 defines an ambiguous asymmetric information economy and introduces the maximin individually rational and ex ante maximin efficient notions. In Sect. 3, we present a counterexample which indicates why the type model cannot capture the case of incompatible reports when agents' preferences are maximin. In Sect. 4, we introduce the direct revelation mechanism and the maximin equilibrium. Sections 5 and 6 present the main results of the paper. Finally, we conclude in Sect. 7.

2 Ambiguous asymmetric information economy

Let Ω denote a finite set of states of nature, $\omega \in \Omega$ a state of nature, $\mathbb{R}^{\ell}_{+}$ the ℓ-goods commodity space and I the set of N agents, i.e., $I = \{1, \ldots, N\}$. *An ambiguous asymmetric information economy* $\mathcal{E}$ is a set $\mathcal{E} = \{\Omega; (\mathcal{F}_i, P_i, e_i, u_i) : i \in I\}$.

$\mathcal{F}_i$ is a partition of Ω. Let $E^{\mathcal{F}_i} \in \mathcal{F}_i$ denote an event and $\omega \in E^{\mathcal{F}_i}$ a state in the event. Then, in the interim, if the state ω occurs, agent i only knows that the event $E^{\mathcal{F}_i}$ has occurred. We impose the standard assumption, that when a state occurs, and all agents truthfully report their information, they will know the realized state.[2] That is,

Assumption 1 For each ω, $\bigcap_{j \in I} E^{\mathcal{F}_j}(\omega) = \{\omega\}$, where $E^{\mathcal{F}_j}(\omega)$ denotes the element in $\mathcal{F}_j$ that contains the state ω.

[2] This assumption is without loss of generality, since if there exist two different states ω and ω', such that no agent is able to distinguish them, then the two states may as well be treated as one state.

Since the events are observable in the interim, it is natural to assume that at ex ante each agent is able to form a probability assessment over his partition. For each i, let $\mu_i : \sigma(\mathcal{F}_i) \to [0, 1]$ be a probability measure defined on the algebra generated by agent i's partition.

Assumption 2 For each i and for each event $E^{\mathcal{F}_i} \in \mathcal{F}_i$, $\mu_i\left(E^{\mathcal{F}_i}\right) > 0$.

Each μ_i is a well-defined probability, but it is not defined on every state of nature. Indeed, if $E^{\mathcal{F}_i} = \left\{\omega, \omega'\right\}$ with $\omega \neq \omega'$, then the probability of the event $E^{\mathcal{F}_i}$ is well defined, but not the probability of the event $\{\omega\}$ or the event $\left\{\omega'\right\}$. Let Δ_i be the set of all probability measures over 2^{Ω} that agree with μ_i. Formally,

$$\Delta_i = \left\{\text{probability measure } \ \pi_i : 2^{\Omega} \to [0, 1] \mid \pi_i(A) = \mu_i(A), \forall A \in \sigma(\mathcal{F}_i)\right\}.$$

Let P_i, a nonempty, closed and convex subset of Δ_i, be agent i's multi-prior set.

Agent i's random initial endowment is $e_i : \Omega \to \mathbb{R}^{\ell}_{+}$. Each agent receives his endowment in the interim. That is, e_i is $\mathcal{F}_i$-measurable, meaning that $e_i(\cdot)$ is constant on each element of $\mathcal{F}_i$. Formally,

Assumption 3 Let $\mathcal{F}_i$ be agent i's partition and fix any $\omega_k \in \Omega$. We have $e_i(\omega) = e_i(\omega_k)$ for any $\omega \in E^{\mathcal{F}_i}(\omega_k)$.

This ensures that at each state ω, the event $E^{\mathcal{F}_i}(\omega)$ incorporates the information revealed by the endowment. Clearly, if each e_i is state independent, then it is automatically $\mathcal{F}_i$-measurable. Assuming e_i to be $\mathcal{F}_i$-measurable is more general than being constant.

Finally, $u_i : \mathbb{R}^{\ell}_{+} \times \Omega \to \mathbb{R}$ is agent i's ex post utility function, taking the form of $u_i(c_i; \omega)$, where c_i denotes agent i's consumption. The ex post utility function u_i is strictly monotone in consumption.[3] Also, we assume that each agent knows his utility function in the interim, and consequently, each agent's utility function needs to be $\mathcal{F}_i$-measurable. Formally,

Assumption 4 For each i and for each fixed $c_i \in \mathbb{R}^{\ell}_{+}$, $u_i(c_i; \cdot)$ is $\mathcal{F}_i$-measurable. That is, given any $c_i \in \mathbb{R}^{\ell}_{+}$, and any two states $\omega, \hat{\omega} \in \Omega$, with $\omega \neq \hat{\omega}$, we have $u_i(c_i; \omega) = u_i\left(c_i; \hat{\omega}\right)$, whenever $\omega \in E^{\mathcal{F}_i}\left(\hat{\omega}\right)$.

The $\mathcal{F}_i$-measurability of the ex post utility functions is often assumed in games with incomplete information. Indeed, one may regard $\mathcal{F}_i$ as agent i's type space, and $E^{\mathcal{F}_i} \in \mathcal{F}_i$ as a possible type of agent i. Then clearly, assuming the function $u_i(c_i; \cdot)$ to be $\mathcal{F}_i$-measurable, is the same as assuming u_i to depend on agent i's type.

Let L denote the set of all functions from Ω to $\mathbb{R}^{\ell}_{+}$. *Agent i's allocation* (or in short, *i-allocation*) specifies his consumption bundle at each state of nature, i.e., $x_i \in L$. Let $x = (x_1, \ldots, x_N)$ denote *an allocation* of the above economy $\mathcal{E}$. An allocation x is said to be *feasible*, if for each $\omega \in \Omega$, $\sum_{i \in I} x_i(\omega) = \sum_{i \in I} e_i(\omega)$.

We postulate that the agents have maximin preferences (see Gilboa and Schmeidler (1989)).

[3] For each fixed ω, we have $u_i(c_i; \omega) < u_i(c_i + \epsilon; \omega)$, whenever ϵ is a none zero vector in $\mathbb{R}^{\ell}_{+}$.

Definition 1 Take any two allocations of agent i, f_i and h_i, from the set L. Agent i prefers f_i to h_i under the maximin preferences (written as $f_i \succeq_i^{MP} h_i$)

$$\min_{\pi_i \in P_i} \sum_{\omega \in \Omega} u_i \left(f_i \left(\omega\right); \omega\right) \pi_i \left(\omega\right) \geq \min_{\pi_i \in P_i} \sum_{\omega \in \Omega} u_i \left(h_i \left(\omega\right); \omega\right) \pi_i \left(\omega\right). \qquad (1)$$

Furthermore, agent i strictly prefers f_i to h_i, $f_i \succ_i^{MP} h_i$, if he prefers f_i to h_i but not the reverse, i.e., $f_i \succeq_i^{MP} h_i$ but $h_i \not\succeq_i^{MP} f_i$.

The Bayesian and the Wald-type maximin preferences of de Castro and Yannelis (2009)[4] are special cases of this general multi-prior model. Indeed, if each agent has a prior, i.e., P_i is a singleton set for each i, then the multi-prior preferences become the Bayesian preferences. If $P_i = \Delta_i$ for each i, then the multi-prior preferences become the maximin preferences in de Castro and Yannelis (2009). They (de Castro and Yannelis 2009) showed that (1) is equivalent to the following formulation

$$\sum_{E^{\mathcal{F}_i} \in \mathcal{F}_i} \left(\min_{\omega \in E^{\mathcal{F}_i}} u_i \left(f_i \left(\omega\right); \omega\right) \right) \mu_i \left(E^{\mathcal{F}_i}\right) \geq \sum_{E^{\mathcal{F}_i} \in \mathcal{F}_i} \left(\min_{\omega \in E^{\mathcal{F}_i}} u_i \left(h_i \left(\omega\right); \omega\right) \right) \mu_i \left(E^{\mathcal{F}_i}\right). \qquad (2)$$

The interest of the maximin preferences in de Castro and Yannelis (2009) comes from that only under these preferences any efficient allocation is incentive compatible.[5] Indeed, under the Bayesian preferences, an efficient allocation may not be incentive compatible as it was shown by Holmström and Myerson (1983).

The notions of individual rationality and efficiency below are standard, except now the preferences are maximin.

Definition 2 A feasible allocation $x = (x_i)_{i \in I}$ is said to be (maximin) individually rational, if for each $i \in I$, $x_i \succeq_i^{MP} e_i$.

Definition 3 A feasible allocation $x = (x_i)_{i \in I}$ is said to be ex ante maximin efficient, if there does not exist another feasible allocation $y = (y_i)_{i \in I}$, such that $y_i \succeq_i^{MP} x_i$ for all i, and $y_i \succ_i^{MP} x_i$ for at least one i.

Remark 1 Some examples of maximin individually rational and ex ante maximin efficient allocations are *maximin core allocations*, *maximin value allocations* and

[4] See also de Castro et al. (2015).

[5] In addition to the fact that there is no longer a conflict between efficiency and incentive compatibility under the maximin preferences, the adoption of these preferences provides new insights and superior outcomes than the Bayesian preferences as it has been shown in Angelopoulos and Koutsougeras (2015), Bodoh-Creed (2012), Bose et al. (2006), de Castro and Yannelis (2009), Castro et al. (2011), de Castro et al. (2015), Gollier (2014), He and Yannelis (2015), Koufopoulos and Kozhan (2016), Liu (2014), Ohtaki and Ozaki (2015), Traeger (2014), just to name a few. Furthermore, the maximin preferences solve the Ellsberg Paradox (see, e.g., Castro and Yannelis (2013)). Interestingly, ambiguity does not necessarily vanish through statistical learning in an one-urn environment (Zimper and Ma (2016)).

maximin Walrasian expectations equilibrium allocations defined in de Castro and Yannelis (2009), Angelopoulos and Koutsougeras (2015), He and Yannelis (2015).

3 A comparison with de Castro–Liu–Yannelis

With the standard type model, de Castro et al. (2015) showed that in an ℓ-goods economy, each maximin individually rational and ex ante maximin efficient allocation can be reached by means of noncooperation. The standard type model is less general than the partition model of this paper. A partition model can be represented by the standard type model if the economy satisfies a stronger assumption than Assumption 1:

Assumption 5 For any $j \in I$ and $E^{\mathcal{F}_j} \in \mathcal{F}_j$, $\bigcap_{j\in I} E^{\mathcal{F}_j} = \{\omega\}$ for some $\omega \in \Omega$.

Assumption 5 ensures that there is always an agreed state, regardless of the agents' reports.[6] Unlike Assumption 1, Assumption 5 does not allow incompatible reports, i.e., the existence of a combination $E^{\mathcal{F}_1}, \ldots, E^{\mathcal{F}_N}$ with $\bigcap_{j\in I} E^{\mathcal{F}_j} = \emptyset$. Clearly, Assumption 5 is not satisfied by many economies. For a detailed discussion on the relationship between partition model and type model, please see de Castro et al. (2016). Below we present a counterexample which indicates why the type model cannot capture the case of incompatible reports when agents are ambiguous. This is not the case for agents with Bayesian preferences. Furthermore, in the economy below, the agents are strictly better off if they trade, and therefore, the problem of implementation is relevant and interesting.

Example 1 There are two agents, one good, and three possible states of nature $\Omega = \{a, b, c\}$. The ex post utility function of each agent i is $u_i(c_i; \omega) = \sqrt{c_i}$. The agents' random initial endowments, information partitions and multi-prior sets are:

$$(e_1(a), e_1(b), e_1(c)) = (5, 5, 1); \quad \mathcal{F}_1 = \{\{a, b\}, \{c\}\}$$
$$(e_2(a), e_2(b), e_2(c)) = (5, 1, 5); \quad \mathcal{F}_2 = \{\{a, c\}, \{b\}\}$$
$$P_1 = \left\{\text{probability measure } \pi_1 : 2^\Omega \to [0, 1] \mid \pi_1(\{a, b\}) = \frac{2}{3} \text{ and } \pi_1(\{c\}) = \frac{1}{3}\right\}.$$
$$P_2 = \left\{\text{probability measure } \pi_2 : 2^\Omega \to [0, 1] \mid \pi_2(\{a, c\}) = \frac{2}{3} \text{ and } \pi_2(\{b\}) = \frac{1}{3}\right\}.$$

A maximin individually rational and ex ante maximin efficient allocation is

$$x = \begin{pmatrix} x_1(a) & x_1(b) & x_1(c) \\ x_2(a) & x_2(b) & x_2(c) \end{pmatrix} = \begin{pmatrix} 5 & 4.8 & 1.2 \\ 5 & 1.2 & 4.8 \end{pmatrix}.$$

[6] Indeed, in the standard type model, each agent i has a type set T_i, and the set of states of nature is $T = T_1 \times \cdots \times T_N$. Let $T_i = F_i$ and $T = \Omega$, then we have $T = T_1 \times \cdots \times T_N$ because of Assumption 5. That is, if a partition model satisfies Assumption 5, then it can be represented by the standard type model.

Notice that both agents strictly prefer the allocation x to the initial endowment e under the maximin preferences. Indeed, we have for each i,

$$\frac{2}{3}\min\left\{\sqrt{5},\sqrt{5}\right\}+\frac{1}{3}\sqrt{1}=1.824<\frac{2}{3}\min\left\{\sqrt{5},\sqrt{4.8}\right\}+\frac{1}{3}\sqrt{1.2}=1.826.$$

This economy cannot be captured by the type model adopted in de Castro et al. (2015). Indeed, in the terminology of a type model, each player has two types, $T_1 = \{\{a, b\}, \{c\}\}$ and $T_2 = \{\{a, c\}, \{b\}\}$. It follows that the set of states of nature $T = T_1 \times T_2$ has four elements. Since there are only three states of nature a, b and c in this economy, it is not possible to represent the economy by the type model, unless we add a zero probability state d. Due to the nonzero probability event assumption (Assumption 1 of de Castro et al. (2015)), we cannot have $\{d\} \in T_i, i = 1, 2$. The only way to incorporate the state d is to have $T_1 = \{\{a, b\}, \{c, d\}\}$ and $T_2 = \{\{a, c\}, \{b, d\}\}$. However, this is not consistent with the non-Bayesian expected utility of this paper. In a Wald-type maximin model, an agent does not form a probability assessment on the states that he cannot distinguish.[7] Since states c and d are in the same event, agent 1 cannot distinguish these two states. However, if agent 1 knows that state d occurs with probability zero, then he can form a unique probability assessment on the states within the event $\{c, d\}$, i.e., $\pi_1(\{c\}) = \frac{1}{3}$ and $\pi_1(\{d\}) = 0$. This contradicts the fact that an ambiguous agent assigns a probability only on an event (and not on the states in the event that he cannot distinguish). A similar argument holds for agent 2 at the event $\{b, d\}$.

The question we pose is the following: Could one provide a noncooperative foundation for the maximin efficient and maximin individually rational notions in a general partition model? We address this question in the next section. Since the type model of de Castro et al. (2015) cannot cover incompatible reports, the proof of de Castro et al. (2015) does not work here. Our work generalizes and extends de Castro et al. (2015).

4 The direct revelation mechanism and the maximin equilibrium

A direct revelation mechanism is a noncooperative game, which is defined based on an allocation and its underlying ambiguous asymmetric information economy.

In the interim, a state of nature ω is realized. Each player i privately observes the event $E^{\mathcal{F}_i}(\omega)$ and receives the initial endowment $e_i(\omega)$. Then, each player i reports $E^{\mathcal{F}_i} \in \mathcal{F}_i$, but the report $E^{\mathcal{F}_i}$ may not be truthful.

Definition 4 Suppose *the realized state* (the true state) is ω. Then, a report of player i, $E^{\mathcal{F}_i} \in \mathcal{F}_i$, is a lie, if it differs from the event $E^{\mathcal{F}_i}(\omega)$.

Definition 5 A *strategy* of player i is a function $s_i : \mathcal{F}_i \to \mathcal{F}_i$.

Let S_i denote player i's strategy set, $S = \times_{i \in I} S_i$ the strategy set and $s \in S$ a strategy profile. For simplicity, we slightly abuse the notation, and use $s(\omega)$ to

[7] In this paper, an agent cannot distinguish the states of nature within an event $E^{\mathcal{F}_i} \in \mathcal{F}_i$.

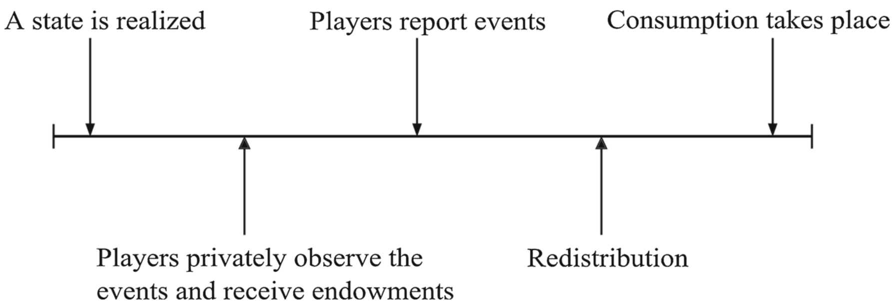

Fig. 1 Timeline

denote the players' reports, when they adopt the strategy profile s, and the realized state is ω, i.e., $s(\omega) = \left(s_1\left(E^{\mathcal{F}_1}(\omega)\right), \ldots, s_N\left(E^{\mathcal{F}_N}(\omega)\right)\right)$. Clearly, for any $\omega \in \Omega$, $s(\omega) \in \times_{i \in I} \mathcal{F}_i$.

The players report events simultaneously. The reports then determine their net transfers. Figure 1 shows the timeline, as in de Castro et al. (2015).

Definition 6 Let x be the planned allocation. *The planned redistribution* (planned net transfer) is given by $x - e$. That is, the planned redistribution is the adjustments needed to go from the initial endowment e to x.

The planned redistribution and the players' reports together determine the actual redistribution. From Assumption 1, we know for any collection of reports $E^{\mathcal{F}_1}, \ldots, E^{\mathcal{F}_N}$, the $\bigcap_{j \in I} E^{\mathcal{F}_j}$ is either singleton or empty.

Definition 7 We say the reports $E^{\mathcal{F}_1}, \ldots, E^{\mathcal{F}_N}$ are compatible, if $\bigcap_{j \in I} E^{\mathcal{F}_j} = \{\tilde{\omega}\}$, for some $\tilde{\omega} \in \Omega$. Furthermore, we refer the state $\tilde{\omega}$ as the implied state (the agreed state).

The implementation literature often assumes a feasibility condition, i.e., the set of feasible alternatives is the same across the states of nature. Now, our players receive initial endowments first, and then redistribute the endowments based on their reports, as in de Castro et al. (2015). We adopt the feasibility condition of de Castro et al. (2015) that each player is rich enough to participate in the revelation mechanism. That is, for each i, ω and $\tilde{\omega}$, we have $e_i(\omega) + x_i(\tilde{\omega}) - e_i(\tilde{\omega}) \in \mathbb{R}^{\ell}_{+}$.

When the reports $E^{\mathcal{F}_1}, \ldots, E^{\mathcal{F}_N}$ are compatible, the players end up with $e(\omega) + x(\tilde{\omega}) - e(\tilde{\omega})$, where $\tilde{\omega}$ is the agreed state, and $x(\tilde{\omega}) - e(\tilde{\omega})$ is the planned redistribution specified for the state $\tilde{\omega}$. Clearly, if all the players tell the truth, then $\tilde{\omega} = \omega$ and the players get what they planned to get, $e(\omega) + x(\omega) - e(\omega) = x(\omega)$. However, since some player may successfully lie, $\tilde{\omega}$ may not be the true state. As a consequence, $e(\omega) + x(\tilde{\omega}) - e(\tilde{\omega})$ may differ from $x(\omega)$, i.e., the players may not end up with the planned allocation.

When the reports are not compatible at the realized state ω, lies are detected. There are many ways to resolve the players' payoffs. In a single good economy, the mechanism designer (MD) could appropriate $\min_{\omega \in \Omega} \{x_i - e_i\}$ from each player i. The MD does not know the realized state, but he knows the planned redistribution

$x - e$, so he knows $\min_{\omega \in \Omega} \{x_i - e_i\}$ for each player i. In an ℓ-goods economy, the mechanism designer could randomly pick a state $\tilde{\omega}$ and enforce the net transfer $x(\tilde{\omega}) - e(\tilde{\omega})$. Furthermore, no trade is also an option, that is, each player keeps his initial endowments. The role of the MD is standard. That is, the MD is not a player in this paper.[8] When a 'punishment' is due according to the rules of the mechanism, the MD does not worry about whether carrying out this 'punishment' is socially optimal or not. As we will show in the proof of Theorem 1, the MD does not need to actually impose the 'punishment.' The threat that he will appropriate $\min_{\omega \in \Omega} \{x_i - e_i\}$ from each player when the reports are incompatible, will induce the players to report truthfully.

Let $D_i\left(x - e, \left(E^{\mathcal{F}_1}, \ldots, E^{\mathcal{F}_N}\right)\right)$ denote *the actual redistribution* of player i. It depends on the planned redistribution $x - e$ and the players' reports. Its exact form will be defined in Sects. 5 and 6.

Definition 8 Let g_i be the outcome function of player i, which depends on the planned redistribution, the reports of the players and the realized state of nature, i.e.,

$$g_i\left(x - e, \left(E^{\mathcal{F}_1}, \ldots, E^{\mathcal{F}_N}\right), \omega\right) = e_i(\omega) + D_i\left(x - e, \left(E^{\mathcal{F}_1}, \ldots, E^{\mathcal{F}_N}\right)\right), \quad (3)$$

where $e_i(\omega) + D_i\left(x - e, \left(E^{\mathcal{F}_1}, \ldots, E^{\mathcal{F}_N}\right)\right)$ is the bundle of the goods, that player i ends up consuming.

Finally, define for each i a final payoff function, which tells us the final payoff that player i ends up. Formally,

Definition 9 Denote by v_i the final payoff function of player i. It takes the form of $v_i\left(x - e, \left(E^{\mathcal{F}_1}, \ldots, E^{\mathcal{F}_N}\right); \omega\right) = u_i\left(e_i(\omega) + D_i\left(x - e, \left(E^{\mathcal{F}_1}, \ldots, E^{\mathcal{F}_N}\right)\right); \omega\right)$.

In what follows, we write $v_i\left(\left(E^{\mathcal{F}_1}, \ldots, E^{\mathcal{F}_N}\right); \omega\right)$ *instead of* $v_i\,(x - e, \left(E^{\mathcal{F}_1}, \ldots, E^{\mathcal{F}_N}\right); \omega)$ *for convenience*.

A direct revelation mechanism associated with a planned allocation x and its underlying ambiguous asymmetric information economy $\mathcal{E} = \left\{\Omega; (\mathcal{F}_i, P_i, e_i, u_i)_{i \in I}\right\}$ is a set $\Gamma = \left\langle I, S, x - e, \{g_i\}_{i \in I}, \{v_i\}_{i \in I}\right\rangle$.

In view of the players' Wald-type maximin preferences, we adopt the maximin equilibrium of de Castro et al. (2015). It says that every player adopts a criterion *a la* Wald Wald (1950). That is, each player maximizes the payoff that takes into account the worst actions of all the other players against him and also the worst state that can occur.

Definition 10 In a direct revelation mechanism $\Gamma = \left\langle I, S, x - e, \{g_i\}_{i \in I}, \{v_i\}_{i \in I}\right\rangle$, a strategy profile $s^* = \left(s_1^*, \ldots, s_N^*\right)$ constitutes a *maximin equilibrium* ($\mathbb{ME}$), if for each player i, his strategy s_i^* maximizes his interim payoff lower bound, that is, the function $s_i^* : \mathcal{F}_i \to \mathcal{F}_i$ satisfies, for each $E^{\mathcal{F}_i} \in \mathcal{F}_i$,

[8] There are interesting papers in which the MD is a player. We refer interested readers to Chakravorty et al. (2006) and Baliga et al. (1997).

$$\min_{E^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i};\ \omega' \in E^{\mathcal{F}_i}} v_i\left(s_i^*\left(E^{\mathcal{F}_i}\right), E^{\mathcal{F}_{-i}}; \omega'\right) \geq \min_{E^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i};\ \omega' \in E^{\mathcal{F}_i}} v_i\left(\hat{E}^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}}; \omega'\right), \tag{4}$$

for all $\hat{E}^{\mathcal{F}_i} \in \mathcal{F}_i$, where $E^{\mathcal{F}_{-i}}$ denotes the reports from all the other players, so $E^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i} = \times_{j \neq i} \mathcal{F}_j$.

The maximin equilibrium has the flavor of the robust control of Hansen and Sargent (2001) in which the decision maker maximizes his payoff taking into account the worst possible model.[9] Unlike the restricted maximin equilibrium notion of Dasgupta et al. (1979), the maximin equilibrium does not need each player to correctly guess his opponents' strategies to reach an equilibrium. Moreover, the maximin equilibrium is unique, whenever truth telling is optimal for each player.[10]

Now, we say an allocation x is implementable, if x can be realized through a maximin equilibrium of the direct revelation mechanism $\Gamma = \left\langle I, S, x - e, \{g_i\}_{i \in I}, \{v_i\}_{i \in I}\right\rangle$.

Let $\mathbb{ME}(\Gamma)$ denote the set of maximin equilibria of the mechanism Γ.

Definition 11 Let x be an allocation of an ambiguous asymmetric information economy $\mathcal{E}$, and $\mathbb{ME}(\Gamma)$ the set of maximin equilibria of the mechanism $\Gamma = \left\langle I, S, x - e, \{g_i\}_{i \in I}, \{v_i\}_{i \in I}\right\rangle$. We say *the allocation x is implementable as a maximin equilibrium of the mechanism* Γ if,

$$\exists s^* \in \mathbb{ME}(\Gamma), \text{ such that } g_i\left(x - e, s^*(\omega), \omega\right) = x_i(\omega),$$

for each $\omega \in \Omega$ and for each $i \in I$.

Definition 12 A strategy profile s is *truth telling*, if for each i, $s_i\left(E^{\mathcal{F}_i}\right) = E^{\mathcal{F}_i}$ for each $E^{\mathcal{F}_i} \in \mathcal{F}_i$. We denote such a strategy profile by s^T.

Remark 2 Clearly, if the mechanism Γ has a truth telling maximin equilibrium, i.e., $s^T \in \mathbb{ME}(\Gamma)$, then the allocation x is implementable as a maximin equilibrium of Γ. Indeed, for each state ω, $s^T(\omega) = \left(E^{\mathcal{F}_1}(\omega), \ldots, E^{\mathcal{F}_N}(\omega)\right)$. It follows that

$$\begin{aligned} g_i\left(x - e, s^T(\omega), \omega\right) &= g_i\left(x - e, \left(E^{\mathcal{F}_1}(\omega), \ldots, E^{\mathcal{F}_N}(\omega)\right), \omega\right) \\ &= e_i(\omega) + D_i\left(x - e, \left(E^{\mathcal{F}_1}(\omega), \ldots, E^{\mathcal{F}_N}(\omega)\right)\right) \\ &= e_i(\omega) + x_i(\omega) - e_i(\omega) = x_i(\omega), \end{aligned}$$

for each $\omega \in \Omega$ and for each $i \in I$, which is the requirement of Definition 11.

Remark 3 The implementation results depend on the payoff rules of the direct revelation mechanism. In a type model, the players' reports are always compatible. de

[9] The robust control approach presumes that decision makers are able to specify the set of possible models, but are either unable or unwilling to form a prior over the forms of model misspecification.

[10] Aryal and Stauber (2014) introduced the notions of ϵ-perfect maxmin equilibrium, perfect maxmin equilibrium and robust sequential equilibrium. Their notions allow players to make small mistakes which is not the case with our notion of maximin equilibrium.

Castro et al. (2015) implements each maximin individually rational and ex ante maximin efficient allocation of an ℓ-goods economy. In this paper, the players' reports may not be compatible. We include the players' net transfers (redistribution) when their reports are not compatible. Interestingly, if these net transfers ('punishments') are too gentle or too crude, then a lie can be profitable. The 'punishments' of this paper are just right to induce truth telling.

In a single good economy, if the planned redistribution for ω is larger than the planned redistribution for $\hat{\omega}$, then a player likes the former more regardless of the realized state, since his utility function is strictly monotone in consumption. It follows that if a player can strictly benefit from lying, then it is possible to Pareto improve the planned allocation by reducing the variation of the planned redistribution across states. This contradicts with the efficiency of the planned allocation. That is, if the planned allocation is efficient, then it is not possible to benefit from lying. However, in an $\ell > 1$ goods economy, the argument for the single good economy does not work. In particular, when a player can benefit from lying, it may not be possible to alter the planned redistribution to achieve a Pareto improvement. Indeed, now a player may like good 1 more than good 2 at state ω, and like good 2 more than good 1 at state ω'. That is, it is possible to benefit from lying, even when the planned allocation is efficient. It follows that we need additional conditions to implement each maximin individually rational and ex ante maximin efficient allocation.

5 Implementation in a single good economy

5.1 Implementation

We show that each maximin individually rational and ex ante maximin efficient allocation x in a single good economy is implementable through its corresponding mechanism $\Gamma = \left\langle I, S, x - e, \{g_i\}_{i \in I}, \{v_i\}_{i \in I} \right\rangle$.

Let $x - e$ denote a planned redistribution, and $\left(E^{\mathcal{F}_1}, \ldots, E^{\mathcal{F}_N}\right)$ a list of reports.

Definition 13 The actual redistribution of player i is given by

$$D_i\left(x - e, \left(E^{\mathcal{F}_1}, \ldots, E^{\mathcal{F}_N}\right)\right) = \begin{cases} x_i\left(\tilde{\omega}\right) - e_i\left(\tilde{\omega}\right) & \text{if } \cap_{j \in I} E^{\mathcal{F}_j} = \{\tilde{\omega}\} \\ \min_{\omega' \in \Omega}\left\{x_i\left(\omega'\right) - e_i\left(\omega'\right)\right\} & \text{if } \cap_{j \in I} E^{\mathcal{F}_j} = \emptyset. \end{cases}$$

That is, when reports are not compatible, the mechanism designer (MD) appropriates $\min_{\omega \in \Omega}\{x_i - e_i\}$ from each player i.

Theorem 1 *Denote by x a maximin individually rational and ex ante maximin efficient allocation of a single good economy and $\mathbb{ME}(\Gamma)$ the set of maximin equilibria of the direct revelation mechanism $\Gamma = \left\langle I, S, x - e, \{g_i\}_{i \in I}, \{v_i\}_{i \in I} \right\rangle$. Then, there exists a truth telling maximin equilibrium s^T, which is the unique maximin equilibrium of the mechanism Γ (i.e., $\left\{s^T\right\} = \mathbb{ME}(\Gamma)$), for which we have $g_i\left(x - e, s^T(\omega), \omega\right) = x_i(\omega)$, for each $\omega \in \Omega$ and for each $i \in I$, i.e., the allocation x is implementable as a maximin equilibrium of its corresponding mechanism Γ.*

Remark 4 Since maximin core allocations, maximin value allocations and maximin Walrasian expectations equilibrium allocations are maximin individually rational and ex ante maximin efficient in a partition model, it follows that they are implementable as a maximin equilibrium.

Remark 5 We differ from the full implementation of Jackson (1991), Palfrey and Srivastava (1987), Palfrey and Srivastava (1989) and Hahn and Yannelis (2001) in that, our players are Wald-type maximin not Bayesian. Furthermore, we do not implement all equilibrium allocations. Instead, we pick an allocation and fully implement the allocation with a mechanism, *a la* Bergemann and Morris (2009). de Castro et al. (2015) showed that the maximin implementation of this paper and the robust implementation of Bergemann and Morris (2009) are different. For further discussion on the relationship between robust implementation and maximin implementation in a general setting, we refer readers to Guo and Yannelis (2015).

5.2 Heuristic proof

Now we provide a heuristic proof by means of an example. A complete proof will be given in the next section.

Example 2 Recall Example 1. There are two agents, one commodity, and three possible states of nature $\Omega = \{a, b, c\}$. The ex post utility function of each agent i is $u_i(c_i; \omega) = \sqrt{c_i}$. The agents' random initial endowments, information partitions and multi-prior sets are:

$$(e_1(a), e_1(b), e_1(c)) = (5, 5, 1); \quad \mathcal{F}_1 = \{\{a, b\}, \{c\}\}$$
$$(e_2(a), e_2(b), e_2(c)) = (5, 1, 5); \quad \mathcal{F}_2 = \{\{a, c\}, \{b\}\}$$
$$P_1 = \left\{\text{probability measure } \pi_1 : 2^{\Omega} \to [0, 1] \mid \pi_1(\{a, b\}) = \frac{2}{3} \text{ and } \pi_1(\{c\}) = \frac{1}{3}\right\}.$$
$$P_2 = \left\{\text{probability measure } \pi_2 : 2^{\Omega} \to [0, 1] \mid \pi_2(\{a, c\}) = \frac{2}{3} \text{ and } \pi_2(\{b\}) = \frac{1}{3}\right\}.$$

A maximin individually rational and ex ante maximin efficient allocation is

$$x = \begin{pmatrix} x_1(a) & x_1(b) & x_1(c) \\ x_2(a) & x_2(b) & x_2(c) \end{pmatrix} = \begin{pmatrix} 5 & 4.8 & 1.2 \\ 5 & 1.2 & 4.8 \end{pmatrix}.$$

Let the planned allocation be x. Then, the planned redistribution $x - e$ is

$$(x_1(a) - e_1(a), x_1(b) - e_1(b), x_1(c) - e_1(c)) = (0, -0.2, 0.2)$$

and

$$(x_2(a) - e_2(a), x_2(b) - e_2(b), x_2(c) - e_2(c)) = (0, 0.2, -0.2).$$

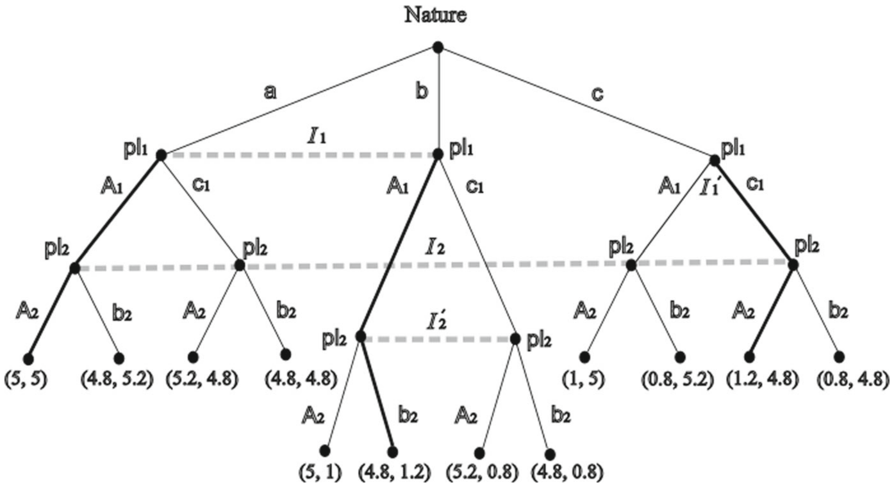

Fig. 2 An informal game tree

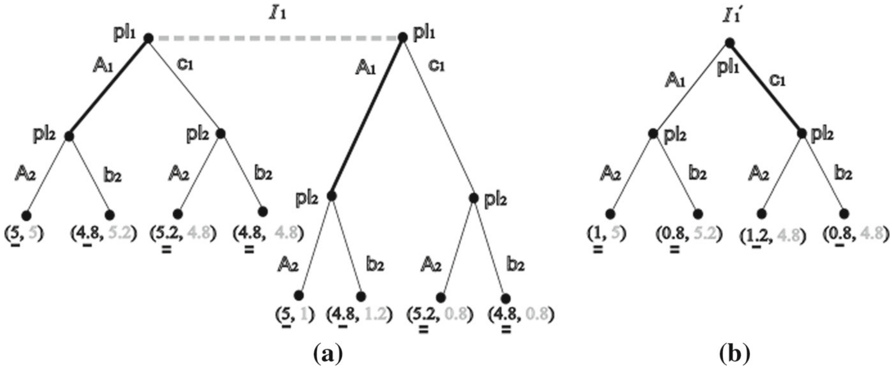

Fig. 3 At player 1's information sets

The game tree is presented in Fig. 2, in which for simplicity, we let $A_1 = \{a, b\}$, $c_1 = \{c\}$, $A_2 = \{a, c\}$, and $b_2 = \{b\}$. We will show that the truth telling strategy profile constitutes the only maximin equilibrium of the game, and the immediate consequence is that the allocation x is implemented. Formally, we will show that the strategy profile $s = (s_1(A_1) = A_1, s_1(c_1) = c_1; s_2(A_2) = A_2, s_2(b_2) = b_2)$, constitutes the only maximin equilibrium of the game.

We look at player 1 first, and she has two information sets $\mathcal{I}_1$ and $\mathcal{I}_1^{'}$ (Fig. 3).

If she is at $\mathcal{I}_1$, then she must have seen the event A_1 from nature. She can either tell the truth A_1 or the lie c_1. Player 1 cannot distinguish the two decision nodes within the set $\mathcal{I}_1$, so her action is common at the two nodes. Figure 3a shows that, being truthful (reports A_1), she may end up with, from the left to the right, 5, 4.8, 5 or 4.8 units of the good. That is, at the information set $\mathcal{I}_1$, if player 1 tells the truth, then she may go down one of the four paths 'aA_1A_2,' 'aA_1b_2,' 'bA_1A_2' and 'bA_1b_2,' for which she ends up with $g_1(x - e, (A_1, A_2), a) = e_1(a) + x_1(a) -$

$e_1(a) = 5 + 0 = 5$, $g_1(x - e, (A_1, b_2), a) = e_1(a) + x_1(b) - e_1(b) = 5 - 0.2 = 4.8$, $g_1(x - e, (A_1, A_2), b) = e_1(b) + x_1(a) - e_1(a) = 5 + 0 = 5$, and $g_1(x - e, (A_1, b_2), b) = e_1(b) + x_1(b) - e_1(b) = 5 - 0.2 = 4.8$ units of the good, respectively. Similarly, by lying (reports c_1), she may end up with, from the left to the right, 5.2, 4.8, 5.2 or 4.8 units of the good.[11] Clearly, when player 1 observes the event A_1, telling the truth (reports A_1) gives her a lower bound payoff of

$$\min\{v_1(A_1, A_2; a), v_1(A_1, b_2; a), v_1(A_1, A_2; b), v_1(A_1, b_2; b)\}$$
$$= \min\left\{\sqrt{5}, \sqrt{4.8}, \sqrt{5}, \sqrt{4.8}\right\} = \sqrt{4.8};$$

lying (reports c_1) gives her a lower bound payoff of

$$\min\{v_1(c_1, A_2; a), v_1(c_1, b_2; a), v_1(c_1, A_2; b), v_1(c_1, b_2; b)\}$$
$$= \min\left\{\sqrt{5.2}, \sqrt{4.8}, \sqrt{5.2}, \sqrt{4.8}\right\} = \sqrt{4.8}.$$

So when she observes the event A_1, she has no incentive to lie, i.e., $s_1(A_1) = A_1$ constitutes part of a maximin equilibrium of the game.

If player 1 is at $\mathcal{I}_1^{'}$, then she must have seen the event c_1 from nature. Figure 3b shows that telling the truth (reports c_1) gives her a lower bound payoff of $\min\{v_1(c_1, A_2; c), v_1(c_1, b_2; c)\} = \min\left\{\sqrt{1.2}, \sqrt{0.8}\right\} = \sqrt{0.8}$; lying (reports A_1) gives her a lower bound payoff of $\min\{v_1(A_1, A_2; c), v_1(A_1, b_2; c)\} = \min\left\{\sqrt{1}, \sqrt{0.8}\right\} = \sqrt{0.8}$. So when she observes the event c_1, she has no incentive to lie, i.e., $s_1(c_1) = c_1$ constitutes part of a maximin equilibrium of the game.

Now, turn to player 2. He has two information sets also, $\mathcal{I}_2$ and $\mathcal{I}_2^{'}$ (Fig. 4).

If player 2 is at $\mathcal{I}_2$, then he must have seen the event A_2 from nature. Figure 4a shows that, being truthful (reports A_2), he may end up with, from the left to the right, 5, 4.8, 5 or 4.8 units of the good; and by lying (reports b_2), he may end up with, from the left to the right, 5.2, 4.8, 5.2 or 4.8 units of the good. Clearly, telling the truth (reports A_2) gives him a lower bound payoff of

$$\min\{v_2(A_1, A_2; a), v_2(c_1, A_2; a), v_2(A_1, A_2; c), v_2(c_1, A_2; c)\}$$
$$= \min\left\{\sqrt{5}, \sqrt{4.8}, \sqrt{5}, \sqrt{4.8}\right\} = \sqrt{4.8};$$

lying (reports b_2) gives him a lower bound payoff of

$$\min\{v_2(A_1, b_2; a), v_2(c_1, b_2; a), v_2(A_1, b_2; c), v_2(c_1, b_2; c)\}$$
$$= \min\left\{\sqrt{5.2}, \sqrt{4.8}, \sqrt{5.2}, \sqrt{4.8}\right\} = \sqrt{4.8}.$$

So when he observes the event A_2, he has no incentive to lie, i.e., $s_2(A_2) = A_2$ constitutes part of a maximin equilibrium of the game.

[11] Notice that both path 'ac_1b_2' and path 'bc_1b_2' lead to incompatible reports. Therefore, the actual redistribution of player 1 is $\min_{\omega' \in \Omega}\left\{x_1(\omega') - e_1(\omega')\right\} = -0.2$.

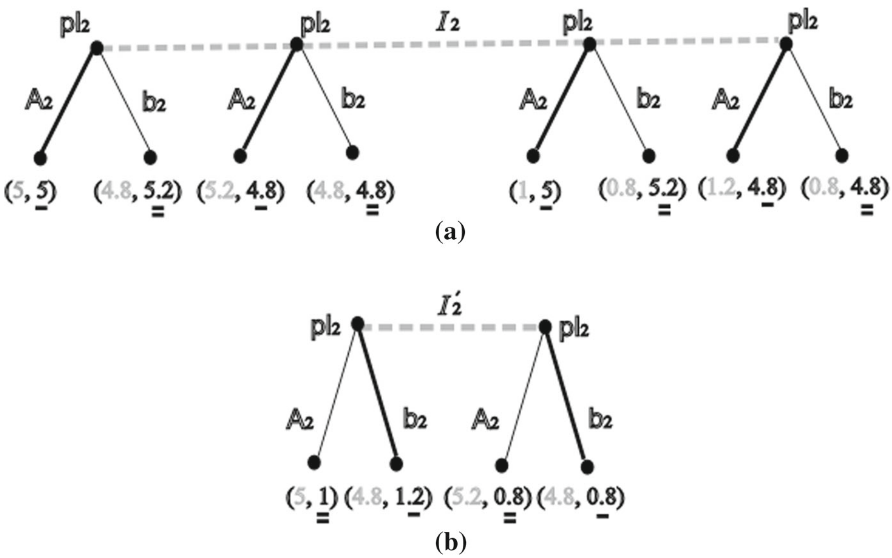

Fig. 4 At player 2's information sets

If player 2 is at $\mathcal{I}_2^{'}$, then he must have seen the event b_2 from nature. Figure 4b shows that telling the truth (reports b_2) gives him a lower bound payoff of $\min\{v_2(A_1, b_2; b), v_2(c_1, b_2; b)\} = \min\left\{\sqrt{1.2}, \sqrt{0.8}\right\} = \sqrt{0.8}$; lying (reports A_2) gives him a lower bound payoff of $\min\{v_2(A_1, A_2; b), v_2(c_1, A_2; b)\} = \min\left\{\sqrt{1}, \sqrt{0.8}\right\} = \sqrt{0.8}$. So when he observes the event b_2, he has no incentive to lie, i.e., $s_2(b_2) = b_2$ constitutes part of a maximin equilibrium of the game.

Now, put together, the strategy profile $s = (s_1(A_1) = A_1, s_1(c_1) = c_1; s_2(A_2) = A_2, s_2(b_2) = b_2)$ is a maximin equilibrium of the game. It is, in fact, the only maximin equilibrium of the game.[12] The equilibrium report paths are $s(a) = (s_1(A_1), s_2(A_2)) = (A_1, A_2)$, $s(b) = (s_1(A_1), s_2(b_2)) = (A_1, b_2)$ and $s(c) = (s_1(c_1), s_2(A_2)) = (c_1, A_2)$, as marked in Fig. 2. It can be easily checked that the maximin individually rational and ex ante maximin efficient allocation x is implemented, since we have $g_1(x - e, s(a), a) = g_1(x - e, (A_1, A_2), a) = 5 + 5 - 5 = 5 = x_1(a)$, and similarly, we have $g_2(x - e, s(a), a) = 5 = x_2(a)$, $g_1(x - e, s(b), b) = 4.8 = x_1(b)$, $g_2(x - e, s(b), b) = 1.2 = x_2(b)$, $g_1(x - e, s(c), c) = 1.2 = x_1(c)$, $g_2(x - e, s(c), c) = 4.8 = x_2(c)$. These outcomes are illustrated in Fig. 2, as pairs following the equilibrium paths.

[12] We assume that a player lies, only if he can benefit from doing so.

5.3 Proof of theorem 1

To ease the explanation, we introduce some notations. We use $\mathcal{F}_{-i} = \times_{j \neq i} \mathcal{F}_j$ to denote the action set of all the players except player i, and $E^{\mathcal{F}_{-i}} = \left(E^{\mathcal{F}_1}, \ldots, E^{\mathcal{F}_{i-1}}, E^{\mathcal{F}_{i+1}}, \ldots, E^{\mathcal{F}_N}\right) \in \mathcal{F}_{-i}$ reports of all the players expect player i.

Furthermore, we write $\omega \in E^{\mathcal{F}_{-i}}$ or $E^{\mathcal{F}_{-i}}(\omega)$, if the state ω belongs to each element in the list $\left(E^{\mathcal{F}_1}, \ldots, E^{\mathcal{F}_{i-1}}, E^{\mathcal{F}_{i+1}}, \ldots, E^{\mathcal{F}_N}\right)$; and we use $E^{\mathcal{F}_i} \bigcap E^{\mathcal{F}_{-i}} = \bigcap_{j \in I} E^{\mathcal{F}_j}$ to denote the information revealed by the reports of all the players.

Let x be a maximin individually rational and ex ante maximin efficient allocation. Suppose that the mechanism Γ does not have a truth telling maximin equilibrium. Then, there must exist a player i, an event $E^{\mathcal{F}_i}$ and a lie $\tilde{E}^{\mathcal{F}_i} \in \mathcal{F}_i$ (clearly, $\tilde{E}^{\mathcal{F}_i} \neq E^{\mathcal{F}_i}$), such that when the player i observes the event $E^{\mathcal{F}_i}$, he can ensure a better lower bound payoff by lying, i.e.,

$$\min_{\substack{E^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i}; \\ \omega' \in E^{\mathcal{F}_i}}} \left\{ v_i \left(E^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}}; \omega' \right) \right\} < \min_{\substack{E^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i}; \\ \omega' \in E^{\mathcal{F}_i}}} \left\{ v_i \left(\tilde{E}^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}}; \omega' \right) \right\}. \tag{5}$$

We will show, in Steps 1 and 2, that (5) cannot hold, and therefore every game Γ has a truth telling maximin equilibrium.

To ease the explanation, denote the left-hand side of (5) by

$$v_i \left(E^{\mathcal{F}_i}, E^*_{-i}; \omega^* \right) = \min_{E^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i};\ \omega' \in E^{\mathcal{F}_i}} \left\{ v_i \left(E^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}}; \omega' \right) \right\},$$

where $E^*_{-i} \in \mathcal{F}_{-i}$ and $\omega^* \in E^{\mathcal{F}_i}$ solve the minimization problem above.

Step 1 We will show that if $E^{\mathcal{F}_i} \cap E^*_{-i} = \{\tilde{\omega}\}$ for some $\tilde{\omega}$, and (5) holds, then x fails to be an ex ante maximin efficient allocation.

Clearly, $\tilde{\omega} \in E^{\mathcal{F}_i}$, and we have

$$\begin{aligned} v_i \left(E^{\mathcal{F}_i}, E^*_{-i}; \omega^* \right) &= u_i \left(e_i \left(\omega^* \right) + x_i \left(\tilde{\omega} \right) - e_i \left(\tilde{\omega} \right); \omega^* \right) = u_i \left(x_i \left(\tilde{\omega} \right); \omega^* \right) \\ &= u_i \left(x_i \left(\tilde{\omega} \right); \tilde{\omega} \right), \end{aligned}$$

as the initial endowment e_i and the utility function u_i are $\mathcal{F}_i$-measurable.

Notice that $v_i \left(E^{\mathcal{F}_i}, E^*_{-i}; \omega^* \right) = u_i \left(x_i \left(\tilde{\omega} \right); \tilde{\omega} \right)$ implies[13]

$$v_i \left(E^{\mathcal{F}_i}, E^*_{-i}; \omega^* \right) = \min_{\omega' \in E^{\mathcal{F}_i}} \left\{ v_i \left(E^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}} \left(\omega' \right); \omega' \right) \right\}. \tag{6}$$

[13] Notice that $\tilde{\omega} \in E^{\mathcal{F}_i}$, and

$$\begin{aligned} v_i \left(E^{\mathcal{F}_i}, E^*_{-i}; \omega^* \right) &= \min_{\substack{E^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i}; \\ \omega' \in E^{\mathcal{F}_i}}} \left\{ v_i \left(E^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}}; \omega' \right) \right\} \leq \min_{\omega' \in E^{\mathcal{F}_i}} \left\{ v_i \left(E^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}} \left(\omega' \right); \omega' \right) \right\} \\ &\leq v_i \left(E^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}} \left(\tilde{\omega} \right); \tilde{\omega} \right) = u_i \left(x_i \left(\tilde{\omega} \right); \tilde{\omega} \right), \end{aligned}$$

imply that we must have equality throughout.

Also, (5) implies[14]

$$v_i\left(E^{\mathcal{F}_i},E^*_{-i};\omega^*\right)<\min_{\omega'\in E^{\mathcal{F}_i}}\left\{v_i\left(\tilde{E}^{\mathcal{F}_i},E^{\mathcal{F}_{-i}}\left(\omega'\right);\omega'\right)\right\}. \tag{7}$$

Now, (6) and (7) together imply

$$\min_{\omega'\in E^{\mathcal{F}_i}}\left\{v_i\left(E^{\mathcal{F}_i},E^{\mathcal{F}_{-i}}\left(\omega'\right);\omega'\right)\right\}<\min_{\omega'\in E^{\mathcal{F}_i}}\left\{v_i\left(\tilde{E}^{\mathcal{F}_i},E^{\mathcal{F}_{-i}}\left(\omega'\right);\omega'\right)\right\}. \tag{8}$$

Finally, Lemma 1 (see below) shows that if (8) holds, then x fails to be an ex ante maximin efficient allocation, which is a contradiction.

Step 2 We will show that if $E^{\mathcal{F}_i}\cap E^*_{-i}=\emptyset$, then (5) cannot hold which is a contradiction. Now we have $v_i\left(E^{\mathcal{F}_i},E^*_{-i};\omega^*\right)=u_i\left(e_i\left(\omega^*\right)+x_i\left(\hat{\omega}\right)-e_i\left(\hat{\omega}\right);\omega^*\right)$, where $\hat{\omega}$ minimizes x_i-e_i. We show by Lemma 2 that

$$\min_{\substack{E^{\mathcal{F}_{-i}}\in\mathcal{F}_{-i};\\ \omega'\in E^{\mathcal{F}_i}}}\left\{v_i\left(\tilde{E}^{\mathcal{F}_i},E^{\mathcal{F}_{-i}};\omega'\right)\right\}=u_i\left(e_i\left(\omega^*\right)+x_i\left(\hat{\omega}\right)-e_i\left(\hat{\omega}\right);\omega^*\right),$$

for every $\tilde{E}^{\mathcal{F}_i}\neq E^{\mathcal{F}_i}$. It follows that

$$\min_{\substack{E^{\mathcal{F}_{-i}}\in\mathcal{F}_{-i};\\ \omega'\in E^{\mathcal{F}_i}}}\left\{v_i\left(E^{\mathcal{F}_i},E^{\mathcal{F}_{-i}};\omega'\right)\right\}=\min_{\substack{E^{\mathcal{F}_{-i}}\in\mathcal{F}_{-i};\\ \omega'\in E^{\mathcal{F}_i}}}\left\{v_i\left(\tilde{E}^{\mathcal{F}_i},E^{\mathcal{F}_{-i}};\omega'\right)\right\}.$$

for every $\tilde{E}^{\mathcal{F}_i}\neq E^{\mathcal{F}_i}$. That is, (5) does not hold.

Therefore, we conclude that the mechanism Γ has a truth telling maximin equilibrium, i.e., $s^T\in\mathbb{ME}(\Gamma)$.

We now show that the truth telling maximin equilibrium is the only maximin equilibrium of the mechanism Γ, i.e., $\left\{s^T\right\}=\mathbb{ME}(\Gamma)$. So suppose otherwise, that is, suppose that both s^T and s^* are maximin equilibria of the mechanism Γ, and $s^T\neq s^*$.

The truth telling strategy profile s^T is different from the strategy profile s^*, which implies that there must exist a player i and an event $E^{\mathcal{F}_i}$, such that

$$s_i^T\left(E^{\mathcal{F}_i}\right)=E^{\mathcal{F}_i}\neq\tilde{E}^{\mathcal{F}_i}=s_i^*\left(E^{\mathcal{F}_i}\right). \tag{9}$$

However, $s_i^*\left(E^{\mathcal{F}_i}\right)=\tilde{E}^{\mathcal{F}_i}\neq E^{\mathcal{F}_i}$ holds, only if lying makes player i strictly better off upon observing the event $E^{\mathcal{F}_i}$, i.e.,

[14] Since by the definition of a minimum, we have that

$$\min_{\substack{E^{\mathcal{F}_{-i}}\in\mathcal{F}_{-i};\\ \omega'\in E^{\mathcal{F}_i}}}\left\{v_i\left(\tilde{E}^{\mathcal{F}_i},E^{\mathcal{F}_{-i}};\omega'\right)\right\}\leq\min_{\omega'\in E^{\mathcal{F}_i}}\left\{v_i\left(\tilde{E}^{\mathcal{F}_i},E^{\mathcal{F}_{-i}}\left(\omega'\right);\omega'\right)\right\}.$$

.

$$\min_{\substack{E^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i};\\ \omega' \in E^{\mathcal{F}_i}}} \left\{ v_i \left(E^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}}; \omega' \right) \right\} < \min_{\substack{E^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i};\\ \omega' \in E^{\mathcal{F}_i}}} \left\{ v_i \left(\tilde{E}^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}}; \omega' \right) \right\},$$

which contradicts to the fact that the truth telling strategy profile constitutes a maximin equilibrium of the mechanism.

Clearly, the maximin individually rational and ex ante maximin efficient allocation x is implemented. Indeed, under the truth telling strategy profile s^T, the list of reports associated to each state ω is

$$s^T(\omega) = \left(E^{\mathcal{F}_1}(\omega), \ldots, E^{\mathcal{F}_N}(\omega) \right).$$

That is, the players always tell the truth. As a consequence, we have

$$\begin{aligned} g_i\left(x - e, s^T(\omega), \omega\right) &= g_i\left(x - e, \left(E^{\mathcal{F}_1}(\omega), \ldots, E^{\mathcal{F}_N}(\omega)\right), \omega\right) \\ &= e_i(\omega) + D_i\left(x - e, \left(E^{\mathcal{F}_1}(\omega), \ldots, E^{\mathcal{F}_N}(\omega)\right)\right) \\ &= e_i(\omega) + x_i(\omega) - e_i(\omega) = x_i(\omega), \end{aligned}$$

for each $\omega \in \Omega$ and for each $i \in I$—the requirement of Definition 11.

Lemma 1 *Given a direct revelation mechanism* $\Gamma = \left\langle I, S, x - e, \{g_i\}_{i \in I}, \{v_i\}_{i \in I} \right\rangle$, *if the allocation* x *is maximin individually rational and ex ante maximin efficient, then there do not exist a player* i, *an event* $E^{\mathcal{F}_i}$ *and a lie* $\tilde{E}^{\mathcal{F}_i} \in \mathcal{F}_i$ *(clearly,* $\tilde{E}^{\mathcal{F}_i} \neq E^{\mathcal{F}_i}$*), such that*[15]

$$\min_{\omega' \in E^{\mathcal{F}_i}} \left\{ v_i \left(E^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}}(\omega'); \omega' \right) \right\} < \min_{\omega' \in E^{\mathcal{F}_i}} \left\{ v_i \left(\tilde{E}^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}}(\omega'); \omega' \right) \right\}. \quad (10)$$

That is, for all i, *given that all the other players tell the truth, it is optimal for player* i *to tell the truth.*[16]

Proof Suppose that there exist a player i, an event $E^{\mathcal{F}_i}$ and a lie $\tilde{E}^{\mathcal{F}_i} \neq E^{\mathcal{F}_i}$, such that (10) holds. We will show that the feasible allocation x fails to be ex ante efficient under the maximin preferences. The idea is similar to the one in theorem 4.1 of de Castro and Yannelis (2009).

Notice that for each $\omega' \in E^{\mathcal{F}_i}$, we have

$$v_i\left(E^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}}(\omega'); \omega'\right) = u_i\left(x_i(\omega'); \omega'\right),$$

and therefore, the left-hand side of (10) can be rewritten as

$$\min_{\omega' \in E^{\mathcal{F}_i}} \left\{ v_i \left(E^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}}(\omega'); \omega' \right) \right\} = \min_{\omega' \in E^{\mathcal{F}_i}} \left\{ u_i\left(x_i(\omega'); \omega'\right) \right\}.$$

[15] In words, (10) says that if all the other players are truthful, then player i can ensure a higher lower bound payoff by lying under the event $E^{\mathcal{F}_i}$.

[16] In other words, truth telling for all i turns out to be a fixed point.

Define an i-allocation of player i, $z_i(\cdot)$, such that for each $\omega' \in E^{\mathcal{F}_i}$,

$$v_i\left(\tilde{E}^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}}(\omega'); \omega'\right) = u_i\left(z_i(\omega'); \omega'\right),$$

and therefore, the right-hand side of (10) can be rewritten as

$$\min_{\omega' \in E^{\mathcal{F}_i}} \left\{v_i\left(\tilde{E}^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}}(\omega'); \omega'\right)\right\} = \min_{\omega' \in E^{\mathcal{F}_i}} \left\{u_i\left(z_i(\omega'); \omega'\right)\right\}.$$

It follows from (10) that

$$\min_{\omega' \in E^{\mathcal{F}_i}} \left\{u_i\left(x_i(\omega'); \omega'\right)\right\} < \min_{\omega' \in E^{\mathcal{F}_i}} \left\{u_i\left(z_i(\omega'); \omega'\right)\right\}, \tag{11}$$

which then implies that

$$\begin{aligned} &\text{for each } \ \omega' \in \arg\min_{\omega'' \in E^{\mathcal{F}_i}} \left\{u_i\left(x_i\left(\omega''\right); \omega''\right)\right\}, \\ &\text{we have } \ u_i\left(x_i(\omega'); \omega'\right) < u_i\left(z_i(\omega'); \omega'\right). \end{aligned} \tag{12}$$

For (12) to hold, it must be the case that for *each* $\omega' \in \arg\min_{\omega'' \in E^{\mathcal{F}_i}} \left\{u_i\left(x_i\left(\omega''\right); \omega''\right)\right\}$, *there exists a state* $\tilde{\omega}$, such that

1. $\tilde{E}^{\mathcal{F}_i} \cap E^{\mathcal{F}_{-i}}(\omega') = \{\tilde{\omega}\}$,
2. $z_i(\omega') = e_i(\omega') + x_i(\tilde{\omega}) - e_i(\tilde{\omega}) \neq x_i(\omega')$.

Let $\{\omega', \tilde{\omega}\}$ denote a set, containing a state $\omega' \in \arg\min_{\omega'' \in E^{\mathcal{F}_i}} \left\{u_i\left(x_i\left(\omega''\right); \omega''\right)\right\}$ and *its* corresponding[17] $\tilde{\omega}$. It follows by 1 above that, for each

$$\omega' \in \arg\min_{\omega'' \in E^{\mathcal{F}_i}} \left\{u_i\left(x_i\left(\omega''\right); \omega''\right)\right\},$$

the set $\{\omega', \tilde{\omega}\}$ is a subset of $E^{\mathcal{F}_{-i}}(\omega')$.

Now, we are ready to define an allocation y that Pareto improves x under the maximin preferences. Define for each $j \in I$, the j-allocation $y_j(\cdot)$ by

$$y_j(\omega') = \begin{cases} z_j(\omega') = e_j(\omega') + x_j(\tilde{\omega}) - e_j(\tilde{\omega}) & \text{if } \omega' \in \arg\min_{\omega'' \in E^{\mathcal{F}_i}} \left\{u_i\left(x_i\left(\omega''\right); \omega''\right)\right\} \\ x_j(\omega') & \text{otherwise.} \end{cases}$$

Notice that the allocation y is feasible.

[17] To avoid confusion, it is worthwhile to re-emphasize that different $\omega' \in \arg\min_{\omega'' \in E^{\mathcal{F}_i}} \left\{u_i\left(x_i\left(\omega''\right); \omega''\right)\right\}$ may be matched with a different $\tilde{\omega}$.

Indeed, for a state $\omega' \notin \arg\min_{\omega'' \in E^{\mathcal{F}_i}} \left\{u_i\left(x_i\left(\omega''\right);\omega''\right)\right\}$, we have

$$\sum_{j\in I} y_j\left(\omega'\right) = \sum_{j\in I} x_j\left(\omega'\right) = \sum_{j\in I} e_j\left(\omega'\right);$$

and for a state $\omega' \in \arg\min_{\omega'' \in E^{\mathcal{F}_i}} \left\{u_i\left(x_i\left(\omega''\right);\omega''\right)\right\}$, we have

$$\sum_{j\in I} y_j\left(\omega'\right) = \sum_{j\in I} z_j\left(\omega'\right) = \sum_{j\in I} e_j\left(\omega'\right) + \sum_{j\in I} x_j\left(\tilde{\omega}\right) - \sum_{j\in I} e_j\left(\tilde{\omega}\right) = \sum_{j\in I} e_j\left(\omega'\right)$$

(recall that x is a feasible allocation at the state $\tilde{\omega}$).

From (12) and the definition of y_i, we have

$$\min_{\omega' \in E^{\mathcal{F}_i}} \left\{u_i\left(y_i\left(\omega'\right);\omega'\right)\right\} > \min_{\omega' \in E^{\mathcal{F}_i}} \left\{u_i\left(x_i\left(\omega'\right);\omega'\right)\right\} \tag{13}$$

under the event $E^{\mathcal{F}_i}$; and for any other event $\hat{E}^{\mathcal{F}_i} \in \mathcal{F}_i$, we have

$$\min_{\omega' \in \hat{E}^{\mathcal{F}_i}} \left\{u_i\left(y_i\left(\omega'\right);\omega'\right)\right\} = \min_{\omega' \in \hat{E}^{\mathcal{F}_i}} \left\{u_i\left(x_i\left(\omega'\right);\omega'\right)\right\}.$$

Therefore, combined with the assumption on $\mu_i(\cdot)$ (Assumption 2), we conclude that, for the player i,

$$\sum_{E\in\mathcal{F}_i} \left(\min_{\omega' \in E_i} u_i\left(y_i\left(\omega'\right);\omega'\right)\right)\mu_i\left(E_i\right) > \sum_{E_i\in\mathcal{F}_i} \left(\min_{\omega' \in E_i} u_i\left(x_i\left(\omega'\right);\omega'\right)\right)\mu_i\left(E_i\right). \tag{14}$$

Here we abuse the notations in (14) slightly, in particular, E_i denotes an arbitrary event in $\mathcal{F}_i$. That is, player i strictly prefers the i-allocation y_i to the i-allocation x_i under the maximin preferences. Now, it remains to show that for any other player $k \neq i$, we have y_k is preferred to x_k under the maximin preferences.

Fix an arbitrary player $k \neq i$, and an arbitrary event that player k may observe, $E^{\mathcal{F}_k} \in \mathcal{F}_k$. Notice that if the event $E^{\mathcal{F}_k}$ contains a state

$$\omega' \in \arg\min_{\omega'' \in E^{\mathcal{F}_i}} \left\{u_i\left(x_i\left(\omega''\right);\omega''\right)\right\},$$

then it contains the set $\left\{\omega', \tilde{\omega}\right\}$. So, by the $\mathcal{F}_k$-measurability of e_k, we have $z_k\left(\omega'\right) = e_k\left(\omega'\right) + x_k\left(\tilde{\omega}\right) - e_k\left(\tilde{\omega}\right) = x_k\left(\tilde{\omega}\right)$. Now, for the event $E^{\mathcal{F}_k}$, define $X_k = \left\{x_k\left(\omega'\right) : \omega' \in E^{\mathcal{F}_k}\right\}$ and $Y_k = \left\{y_k\left(\omega'\right) : \omega' \in E^{\mathcal{F}_k}\right\}$. We have $Y_k \subset X_k$. Indeed, if $\omega' \in E^{\mathcal{F}_k}$ and $\omega' \in \arg\min_{\omega'' \in E^{\mathcal{F}_i}} \left\{u_i\left(x_i\left(\omega''\right);\omega''\right)\right\}$, then

$$y_k\left(\omega'\right) = z_k\left(\omega'\right) = x_k\left(\tilde{\omega}\right) \in X_k;$$

and if $\omega' \in E^{\mathcal{F}_k}$ and $\omega' \notin \arg\min_{\omega'' \in E^{\mathcal{F}_i}} \left\{u_i\left(x_i\left(\omega''\right); \omega''\right)\right\}$, then

$$y_k\left(\omega'\right) = x_k\left(\omega'\right) \in X_k.$$

Therefore, with Assumption 4, we have that

$$\min_{\omega' \in E^{\mathcal{F}_k}} \left\{u_k\left(y_k\left(\omega'\right); \omega'\right)\right\} \geq \min_{\omega' \in E^{\mathcal{F}_k}} \left\{u_k\left(x_k\left(\omega'\right); \omega'\right)\right\}.$$

Since the event $E^{\mathcal{F}_k} \in \mathcal{F}_k$ is arbitrary, we conclude that

$$\sum_{E^{\mathcal{F}_k} \in \mathcal{F}_k} \left(\min_{\omega' \in E^{\mathcal{F}_k}} u_k\left(y_k\left(\omega'\right); \omega'\right)\right) \mu_k\left(E^{\mathcal{F}_k}\right)$$
$$\geq \sum_{E^{\mathcal{F}_k} \in \mathcal{F}_k} \left(\min_{\omega' \in E^{\mathcal{F}_k}} u_k\left(x_k\left(\omega'\right); \omega'\right)\right) \mu_k\left(E^{\mathcal{F}_k}\right).$$

Also, since player $k \neq i$ is arbitrary, we have for every player $k \neq i$, y_k is preferred to x_k under the maximin preferences.

Thus, the feasible allocation y Pareto improves the allocation x under the maximin preferences, i.e., x fails to be an ex ante maximin efficient allocation. This contradiction completes the proof of Lemma 1. □

Lemma 2 *Let*

$$v_i\left(E^{\mathcal{F}_i}, E^*_{-i}; \omega^*\right) = \min_{E^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i};\ \omega' \in E^{\mathcal{F}_i}} \left\{v_i\left(E^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}}; \omega'\right)\right\}.$$

If $E^{\mathcal{F}_i} \cap E^*_{-i} = \emptyset$, *then*

$$\min_{\substack{E^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i};\\ \omega' \in E^{\mathcal{F}_i}}} \left\{v_i\left(E^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}}; \omega'\right)\right\} = \min_{\substack{E^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i};\\ \omega' \in E^{\mathcal{F}_i}}} \left\{v_i\left(\tilde{E}^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}}; \omega'\right)\right\}, \tag{15}$$

for each $\tilde{E}^{\mathcal{F}_i} \in \mathcal{F}_i$.

Proof By construction, a player anticipates the worst net transfer in the case of incompatible reports. Since $E^{\mathcal{F}_i} \cap E^*_{-i} = \emptyset$, it follows that $v_i\left(E^{\mathcal{F}_i}, E^*_{-i}; \omega^*\right) = u_i\left(e_i\left(\omega^*\right) + x_i\left(\hat{\omega}\right) - e_i\left(\hat{\omega}\right); \omega^*\right)$, where $\hat{\omega} \in \arg\min_{\omega \in \Omega}\{x_i - e_i\}$. Clearly, given a $\tilde{E}^{\mathcal{F}_i} \in \mathcal{F}_i$ and $\tilde{E}^{\mathcal{F}_i} \neq E^{\mathcal{F}_i}$, if there exists a $\tilde{E}^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i}$ such that $\tilde{E}^{\mathcal{F}_i} \cap \tilde{E}^{\mathcal{F}_{-i}} = \emptyset$, then by Assumptions 3 and 4, we have

$$\min_{\substack{E^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i};\\ \omega' \in E^{\mathcal{F}_i}}} \left\{v_i\left(\tilde{E}^{\mathcal{F}_i}, E^{\mathcal{F}_{-i}}; \omega'\right)\right\} = u_i\left(e_i\left(\omega^*\right) + x_i\left(\hat{\omega}\right) - e_i\left(\hat{\omega}\right); \omega^*\right).$$

That is, (15) holds in this case.

Now, suppose that there exists $\tilde{E}^{\mathcal{F}_i} \neq E^{\mathcal{F}_i}$ such that $\tilde{E}^{\mathcal{F}_i}$ is compatible with all $E^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i}$. We will show that $\arg\min_{\omega\in\Omega} \{x_i - e_i\} \cap \tilde{E}^{\mathcal{F}_i}$ is nonempty. Then, since any state in $\tilde{E}^{\mathcal{F}_i}$ can be an agreed state, player i can end up with the same net transfer as when the reports are not compatible. That is, (15) holds.

Assume otherwise, i.e., assume that $\arg\min_{\omega\in\Omega} \{x_i - e_i\} \cap \tilde{E}^{\mathcal{F}_i} = \emptyset$. It follows that $\arg\min_{\omega\in\Omega} \{x_i - e_i\} \neq \Omega$. Now define an allocation y. For each $\omega' \in \arg\min_{\omega\in\Omega} \{x_i - e_i\}$, let $y\left(\omega'\right) = e\left(\omega'\right) + x\left(\tilde{\omega}\right) - e\left(\tilde{\omega}\right)$, where $\{\tilde{\omega}\} = E^{\mathcal{F}_{-i}}\left(\omega'\right) \cap \tilde{E}^{\mathcal{F}_i}$. Notice, $\tilde{\omega}$ is well defined, since by construction $\tilde{E}^{\mathcal{F}_i}$ is compatible with all $E^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i}$. Also notice that, for each $\omega' \in \arg\min_{\omega\in\Omega} \{x_i - e_i\}$, we have $u_i\left(e_i\left(\omega''\right) + x_i\left(\omega'\right) - e_i\left(\omega'\right); \omega''\right) < u_i\left(e_i\left(\omega''\right) + x_i\left(\tilde{\omega}\right) - e_i\left(\tilde{\omega}\right); \omega''\right)$ for each $\omega'' \in \Omega$, since $\tilde{\omega} \in \tilde{E}^{\mathcal{F}_i}$ (i.e., $\tilde{\omega} \notin \arg\min_{\omega\in\Omega} \{x_i - e_i\}$), and u_i is strictly monotone. Now, for each $\omega' \notin \arg\min_{\omega\in\Omega} \{x_i - e_i\}$, let $y\left(\omega'\right) = x\left(\omega'\right)$.

Clearly, the allocation y is feasible. Also, y Pareto improves x under the maximin preferences. Indeed, at each $\omega' \in \arg\min_{\omega\in\Omega} \{x_i - e_i\}$, we have

$$\min_{\omega''\in E^{\mathcal{F}_i}(\omega')} u_i\left(y_i\left(\omega''\right); \omega'\right) > \min_{\omega''\in E^{\mathcal{F}_i}(\omega')} u_i\left(x_i\left(\omega''\right); \omega'\right).$$

At each ω' with $E^{\mathcal{F}_i}\left(\omega'\right) \cap \arg\min_{\omega\in\Omega} \{x_i - e_i\} = \emptyset$, we have

$$\min_{\omega''\in E^{\mathcal{F}_i}(\omega')} u_i\left(y_i\left(\omega''\right); \omega'\right) = \min_{\omega''\in E^{\mathcal{F}_i}(\omega')} u_i\left(x_i\left(\omega''\right); \omega'\right).$$

Combined with Assumption 2, y_i gives player i a strictly higher ex ante maximin payoff, i.e., player i strictly prefers y_i to x_i at ex ante under the maximin preferences. Let j be an arbitrary player different from i. By construction, for each $\omega' \in \arg\min_{\omega\in\Omega} \{x_i - e_i\}$, its corresponding $\tilde{\omega}$ is in the set $E^{\mathcal{F}_{-i}}\left(\omega'\right)$. That is, player j cannot distinguish ω' and its corresponding $\tilde{\omega}$. By the same argument as in Lemma 1, at each $\omega' \in \arg\min_{\omega\in\Omega} \{x_i - e_i\}$, we have

$$\min_{\omega''\in E^{\mathcal{F}_j}(\omega')} u_j\left(y_j\left(\omega''\right); \omega'\right) \geq \min_{\omega''\in E^{\mathcal{F}_j}(\omega')} u_j\left(x_j\left(\omega''\right); \omega'\right).$$

At each ω' with $E^{\mathcal{F}_j}\left(\omega'\right) \cap \arg\min_{\omega\in\Omega} \{x_i - e_i\} = \emptyset$, we have

$$\min_{\omega''\in E^{\mathcal{F}_j}(\omega')} u_j\left(y_j\left(\omega''\right); \omega'\right) = \min_{\omega''\in E^{\mathcal{F}_j}(\omega')} u_j\left(x_j\left(\omega''\right); \omega'\right).$$

Combined with Assumption 2, player j prefers y_j to x_j at ex ante under the maximin preferences. Therefore, the allocation x is not ex ante maximin efficient. This contradiction completes the proof of Lemma 2. □

6 Implementation in an economy with more than one good

In an economy with more than one good, the actual redistribution (Definition 13) of Sect. 5 is not well defined. We need a new payout rule. Let $x - e$ denote a planned redistribution and $\left(E^{\mathcal{F}_1}, \ldots, E^{\mathcal{F}_N}\right)$ a list of reports.

Definition 14 The actual redistribution of player i is given by

$$D_i\left(x-e,\left(E^{\mathcal{F}_1},\ldots,E^{\mathcal{F}_N}\right)\right)=\begin{cases} x_i\left(\tilde{\omega}\right)-e_i\left(\tilde{\omega}\right) & \text{if } \cap_{j\in I} E^{\mathcal{F}_j}=\{\tilde{\omega}\} \\ x_i\left(\hat{\omega}\right)-e_i\left(\hat{\omega}\right) \text{ for some } \hat{\omega}, & \text{if } \cap_{j\in I} E^{\mathcal{F}_j}=\emptyset. \end{cases}$$

Now, whenever the players' reports are not compatible, the mechanism designer (MD) carries out a net transfers $x\left(\hat{\omega}\right) - e\left(\hat{\omega}\right)$. The $\hat{\omega}$ is unknown to the players, when they report events.

Being maximin, the players anticipate the worst. That is, for each i and $E^{\mathcal{F}_i}(\omega)$, player i anticipates the payoff[18]

$$\min_{\omega'\in\Omega} u_i\left(e_i\left(\omega\right)+x_i\left(\omega'\right)-e_i\left(\omega'\right);\omega\right) \tag{16}$$

in the case of incompatible reports. Clearly, depending on the event player i observes, he may associate different worst case scenario in the case of incompatible reports. As the proof of our implementation result will indicate, the condition (16) works as a threat which induces players not to deviate from reporting the true events.

6.1 Implementation

We show that each maximin individually rational and ex ante maximin efficient allocation is implementable as a maximin equilibrium, as long as one of the following three conditions holds.

Condition 1 *The economy satisfies Assumption 5 (This is the case studied by de Castro et al. (2015)).*

Condition 2 *If incompatible reports can occur when a player tells the truth, then incompatible reports can occur when he tells a lie. Formally, let ω be the realized state of nature. If there exists $E^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i}$, such that $E^{\mathcal{F}_i}(\omega) \cap E^{\mathcal{F}_{-i}} = \emptyset$, then for every $\tilde{E}^{\mathcal{F}_i} \neq E^{\mathcal{F}_i}(\omega)$, there exists $\tilde{E}^{\mathcal{F}_{-i}} \in \mathcal{F}_{-i}$, such that $\tilde{E}^{\mathcal{F}_i} \cap \tilde{E}^{\mathcal{F}_{-i}} = \emptyset$.*

Condition 3 *Allocation x is a maximin individually rational and ex ante maximin efficient allocation. For each i, the set M_i is not empty, where*

$$M_i = \left\{\omega^m : u_i\left(e_i\left(\omega\right)+x_i\left(\omega^m\right)-e_i\left(\omega^m\right),\omega\right) \leq \right.$$
$$\left. u_i\left(e_i\left(\omega\right)+x_i\left(\tilde{\omega}\right)-e_i\left(\tilde{\omega}\right),\omega\right) \quad \textit{for all} \quad \omega, \tilde{\omega}\in\Omega\right\}.$$

[18] Recall that both the initial endowment and the utility function are $\mathcal{F}_i$-measurable.

Condition 3 ensures that each player has a least preferred planned redistribution.

Remark 6 In the single good case, Condition 3 is automatically satisfied. Indeed, for each i, the set $M_i = \arg\min_{\omega \in \Omega} \{x_i - e_i\}$ which is not empty. It follows that the implementation results of the single good case requires no additional condition.

Theorem 2 *Denote by x a maximin individually rational and ex ante maximin efficient allocation in an economy with more than one good. Its corresponding direct revelation mechanism* $\Gamma = \left\langle I, S, x - e, \{g_i\}_{i \in I}, \{v_i\}_{i \in I} \right\rangle$ *has a truth telling maximin equilibrium* s^T*, which is the unique maximin equilibrium of the mechanism* Γ*, if one of the three conditions (Conditions 1, 2 and 3) holds.*

Proof Condition 1 rules out the case of Step 2 which is in the proof of Theorem 1. Therefore, the result follows from Step 1 and Lemma 1. See also de Castro et al. (2015).

If an economy satisfies condition 2, the result follows from the proof of Theorem 1 and Lemma 1. The case in Step 2 (i.e., $E^{\mathcal{F}_i} \cap E^*_{-i} = \emptyset$) of the proof of Theorem 1 can occur, but condition 2 guarantees that in this case no lie can give the player a strictly higher maximin payoff.

Finally, if an economy satisfies condition 3, then we can replace the state $\hat{\omega}$ in the Step 2 of the proof of Theorem 1 with an $\omega^m \in M_i$, and replace the set $\arg\min_{\omega \in \Omega} \{x_i - e_i\}$ in Lemma 2 with the set M_i. Now, the implementation result follows from the proof of Theorem 1, Lemma 1 and Lemma 2. □

6.2 Counterexamples

Example 3 below shows that if all three conditions of Theorem 2 fail, then a maximin individually rational and ex ante maximin efficient allocation may not be implementable in Γ.

Example 3 There are two agents, $I = \{1, 2\}$, two goods, and six states of nature $\Omega = \{a, b, c, d, e, f\}$. The ex post utility function of agent 1 is $u_1\left(x_{1,1}(\omega), x_{1,2}(\omega); \omega\right) = \sqrt{x_{1,1}(\omega)} + \sqrt{x_{1,2}(\omega)}$, $\omega \in \{a, b, c, d, e\}$, and $u_1\left(x_{1,1}(f), x_{1,2}(f); f\right) = 0.5\sqrt{x_{1,1}(f)} + 2\sqrt{x_{1,2}(f)}$, where the second index refers to the good. The ex post utility function of agent 2 is $u_2\left(x_{2,1}(\omega), x_{2,2}(\omega); \omega\right) = \sqrt{x_{2,1}(\omega)} + \sqrt{x_{2,2}(\omega)}$, $\omega \in \{a, c, e, f\}$, and $u_2\left(x_{2,1}(\omega), x_{2,2}(\omega); \omega\right) = 1.01\sqrt{x_{2,1}(\omega)} + \sqrt{x_{2,2}(\omega)}$, $\omega \in \{b, d\}$. The agents' random initial endowments, information partitions and multi-prior sets are: $e_1(\omega) = (10, 10)$ for all ω; $e_2(\omega) = (10, 10)$ for $\omega \in \{a, b, c, d\}$; and $e_2(\omega) = (11, 9)$ for $\omega \in \{e, f\}$.

$$\mathcal{F}_1 = \{\{a, b\}, \{c, d, e\}, \{f\}\}; \quad \mathcal{F}_2 = \{\{a, c\}, \{b, d\}, \{e, f\}\}.$$

$$P_1 = \left\{\text{probability measure } \pi_1 : 2^\Omega \to [0, 1] \mid \pi_1(\{a, b\}) = \pi_1(\{c, d, e\}) = \pi_1(\{f\}) = \frac{1}{3}\right\}.$$

$$P_2 = \left\{\text{probability measure } \pi_2 : 2^\Omega \to [0, 1] \mid \pi_2(\{a, c\}) = \pi_2(\{b, d\}) = \pi_2(\{e, f\}) = \frac{1}{3}\right\}.$$

A maximin individually rational and ex ante maximin efficient allocation is

$$x = \begin{pmatrix} x_1(a) & x_1(b) & x_1(c) & x_1(d) & x_1(e) & x_1(f) \\ x_2(a) & x_2(b) & x_2(c) & x_2(d) & x_2(e) & x_2(f) \end{pmatrix}$$
$$= \begin{pmatrix} (10, 10) & (9.950311, 10.049813) & (10, 10) & (9.99, 10) & (10.5, 9.5) & (3.469, 14.439) \\ (10, 10) & (10.049689, 9.950187) & (10, 10) & (10.01, 10) & (10.5, 9.5) & (17.531, 4.561) \end{pmatrix}.$$

This example does not satisfy any of the three conditions of Theorem 2. It turns out that the mechanism Γ has a unique maximin equilibrium, in which player 1 (agent 1) lies when he is in the event $\{a, b\}$,[19] and player 2 (agent 2) always reports the true event. That is, $s_1(\{a, b\}) = s_1(\{c, d, e\}) = \{c, d, e\}$, $s_1(\{f\}) = \{f\}$, $s_2(\{a, c\}) = \{a, c\}$, $s_2(\{b, d\}) = \{b, d\}$, $s_2(\{e, f\}) = \{e, f\}$ constitute the unique maximin equilibrium. For each i and ω, let $y_i(\omega) = g_i(x - e, s(\omega), \omega)$. That is, the allocation y is realized through the unique maximin equilibrium:

$$y = \begin{pmatrix} y_1(a) & y_1(b) & y_1(c) & y_1(d) & y_1(e) & y_1(f) \\ y_2(a) & y_2(b) & y_2(c) & y_2(d) & y_2(e) & y_2(f) \end{pmatrix}$$
$$= \begin{pmatrix} (10, 10) & (9.99, 10) & (10, 10) & (9.99, 10) & (10.5, 9.5) & (3.469, 14.439) \\ (10, 10) & (10.01, 10) & (10, 10) & (10.01, 10) & (10.5, 9.5) & (17.531, 4.561) \end{pmatrix}.$$

Clearly, the allocation y differs from x. That is, the allocation x is not implemented.

However, the three conditions of Theorem 2 are not necessary. The following example does not satisfy any of the three conditions, and its maximin individually rational and ex ante maximin efficient allocation is implementable.

Example 4 There are two agents, $I = \{1, 2\}$, two goods, and three states of nature $\Omega = \{a, b, c\}$. The ex post utility function of agent 1 is $u_1\left(x_{1,1}(\omega), x_{1,2}(\omega); \omega\right) = \sqrt{x_{1,1}(\omega)} + 2\sqrt{x_{1,2}(\omega)}$, for all ω, where the second index refers to the good. The ex post utility function of agent 2 is $u_2\left(x_{2,1}(\omega), x_{2,2}(\omega); \omega\right) =$

[19] When player 1 is in the event $\{a, b\}$, the worst net transfer for him is $x_1(f) - e_1(f)$. He may get the worst transfer if the players' reports are not compatible, or if the agreed state is f. The lie $\{c, d, e\}$ allows him to avoid the worst net transfer, and gives him the highest lower bound payoff. Indeed, reporting $\{a, b\}$ gives him a payoff of

$$\begin{aligned} &\min\{v_1(\{a, b\}, \{a, c\}; a), v_1(\{a, b\}, \{b, d\}; a), v_1(\{a, b\}, \{e, f\}; a), \\ &\quad v_1(\{a, b\}, \{a, c\}; b), v_1(\{a, b\}, \{b, d\}; b), v_1(\{a, b\}, \{e, f\}; b)\} \\ &= \min\{6.32455, 6.32455, 5.66239, 6.32455, 6.32455, 5.66239\} = 5.66239; \end{aligned}$$

reporting $\{c, d, e\}$ gives him a payoff of

$$\min\{6.32455, 6.32297, 6.32258, 6.32455, 6.32297, 6.32258\} = 6.32258;$$

and reporting $\{f\}$ gives him a payoff of

$$\min\{5.66239, 5.66239, 5.66239, 5.66239, 5.66239, 5.66239\} = 5.66239.$$

Clearly, the payoff of reporting $\{c, d, e\}$ is the highest.

$2\sqrt{x_{2,1}(\omega)} + \sqrt{x_{2,2}(\omega)}$, for all ω. The agents' random initial endowments, information partitions and multi-prior sets are:

$$(e_1(a), e_1(b), e_1(c)) = ((8,8), (10,2), (10,2)); \quad \mathcal{F}_1 = \{\{a\}, \{b,c\}\}$$
$$(e_2(a), e_2(b), e_2(c)) = ((4,8), (4,8), (15,6)); \quad \mathcal{F}_2 = \{\{a,b\}, \{c\}\}$$
$$P_1 = \left\{\text{probability measure } \pi_1 : 2^\Omega \to [0,1] \mid \pi_1(\{a\}) = \pi_1(\{b,c\}) = \frac{1}{2}\right\}.$$
$$P_2 = \left\{\text{probability measure } \pi_2 : 2^\Omega \to [0,1] \mid \pi_2(\{a,b\}) = \pi_2(\{c\}) = \frac{1}{2}\right\}.$$

A maximin individually rational and ex ante maximin efficient allocation is

$$x = \begin{pmatrix} x_1(a) & x_1(b) & x_1(c) \\ x_2(a) & x_2(b) & x_2(c) \end{pmatrix} = \begin{pmatrix} (2.4, 12.8) & (2.8, 8) & (5, 6.4) \\ (9.6, 3.2) & (11.2, 2) & (20, 1.6) \end{pmatrix}.$$

This example does not satisfy any of the three conditions of Theorem 2. It turns out that the mechanism Γ has a unique maximin equilibrium, in which the players (agents 1 and 2) always report the true event. That is, $s_1(\{a\}) = \{a\}$, $s_1(\{b,c\}) = \{b,c\}$, $s_2(\{a,b\}) = \{a,b\}$, $s_2(\{c\}) = \{c\}$ constitute the unique maximin equilibrium. Consequently, the allocation x is realized through the unique maximin equilibrium.

6.3 The case of state-independent linear ex post utility functions

To assume linear ex post utility functions is a rather strong assumption, but in this case no additional assumption is needed for implementation in an ℓ-goods economy.

Corollary 1 *Each maximin individually rational and ex ante maximin efficient allocation x is implementable as a maximin equilibrium in an ℓ-goods economy*[20] *with state-independent linear ex post utility functions, i.e., for each i and ω, $u_i(c_i;\omega) = \sum_{k=1}^{\ell} \alpha_k c_i^k$. That is, its corresponding direct revelation mechanism $\Gamma = \langle I, S, x-e, \{g_i\}_{i\in I}, \{v_i\}_{i\in I}\rangle$ has a truth telling maximin equilibrium s^T, which is the unique maximin equilibrium of the mechanism Γ.*

Proof If $\ell = 1$, Corollary 1 follows immediately from Theorem 1. We look into the case of $\ell > 1$. Let x be a maximin individually rational and ex ante maximin efficient allocation, and let $x - e$ be its corresponding planned redistribution.

At state ω, if the actual redistribution is $x(\hat{\omega}) - e(\hat{\omega})$, then player i's ex post utility is

[20] Assumption 1 is assumed instead of Assumption 5.

$$u_i\left(e_i(\omega)+x_i\left(\hat{\omega}\right)-e_i\left(\hat{\omega}\right);\omega\right)=\sum_{k=1}^{\ell}\alpha_k\left[e_i^k(\omega)+x_i^k\left(\hat{\omega}\right)-e_i^k\left(\hat{\omega}\right)\right]$$
$$=\sum_{k=1}^{\ell}\alpha_k e_i^k(\omega)+\sum_{k=1}^{\ell}\alpha_k\left[x_i^k\left(\hat{\omega}\right)-e_i^k\left(\hat{\omega}\right)\right]. \quad (17)$$

Let $\sum_{k=1}^{\ell}\alpha_k\left[x_i^k(\omega^*)-e_i^k(\omega^*)\right] = \min\left\{\sum_{k=1}^{\ell}\alpha_k\left[x_i^k\left(\hat{\omega}\right)-e_i^k\left(\hat{\omega}\right)\right]:\hat{\omega}\in\Omega\right\}$. Clearly, for all ω, $x_i(w^*)-e_i(w^*)$ is player i's least preferred planned redistribution. It follows that

$$\omega^*\in M_i=\left\{\omega^m : u_i\left(e_i(\omega)+x_i\left(\omega^m\right)-e_i\left(\omega^m\right),\omega\right)\leq u_i\left(e_i(\omega)+x_i(\tilde{\omega})-e_i(\tilde{\omega}),\omega\right) \quad \text{for all} \quad \omega,\tilde{\omega}\in\Omega\right\}.$$

Also, since i is arbitrary, we have for each i, the set M_i is not empty. That is, Condition 3 holds. Now, it follows from Theorem 2 that the direct revelation mechanism $\Gamma=\left\langle I,S,x-e,\{g_i\}_{i\in I},\{v_i\}_{i\in I}\right\rangle$ has a truth telling maximin equilibrium s^T, which is the unique maximin equilibrium of the mechanism Γ. That is, x is implementable as a maximin equilibrium. □

7 Concluding remarks

We introduce conditions under which each maximin individually rational and ex ante maximin efficient allocation is implementable by means of noncooperative behavior under ambiguity. That is, any arbitrary maximin individually rational and ex ante maximin efficient allocation x can be reached through the mechanism $\Gamma = \left\langle I,S,x-e,\{g_i\}_{i\in I},\{v_i\}_{i\in I}\right\rangle$ as its unique maximin equilibrium outcome.

In particular, we show that each maximin individually rational and ex ante maximin efficient allocation is implementable in a single good economy. These allocations are implementable in a multi-goods economy, provided that at least one of the three sufficient conditions in Theorem 2 is satisfied. Furthermore, if the agents have state-independent linear ex post utility functions, then each maximin individually rational and ex ante maximin efficient allocation is implementable in a multi-goods economy.

Our implementation results depend on the payoff rules of the direct revelation mechanism. In particular, we include the players' net transfers (redistribution) when their reports are not compatible. We assume that the players associate incompatible reports with the worst planned net transfer, i.e., we impose a 'punishment' whenever the reports are not compatible. The threat of a 'punishment' by the mechanism designer induces truthful reports. It is an open question, whether altering the payoff rules one can make each maximin individually rational and ex ante maximin efficient allocation implementable in a multi-goods economy. Changing our 'punishment' to 'no trade'

[21] does not help. In particular, the allocation of Example 3 is not implementable under the 'no trade' rule. Furthermore, it follows from our proofs that if the mechanism designer knows the realized state of nature ω in the interim,[22] and carries out the correct planned net transfer $x(\omega) - e(\omega)$ whenever the reports are not compatible, then each maximin individually rational and ex ante maximin efficient allocation is implementable.

References

Angelopoulos, A., Koutsougeras, L.C.: Value allocation under ambiguity. Econ. Theory **59**(1), 147–167 (2015)

Aryal, G., Stauber, R.: Trembles in extensive games with ambiguity averse player. Econ. Theory **57**(1), 1–40 (2014)

Baliga, S., Corchón, L., Sjöström, T.: The theory of implementation when the planner is a player. J. Econ. Theory **77**, 15–33 (1997)

Bergemann, D., Morris, S.: Robust implementation in direct mechanisms. Rev. Econ. Stud. **76**, 1175–1204 (2009)

Bodoh-Creed, A.: Ambiguous beliefs and mechanism design. Games Econ. Behav. **75**, 518–537 (2012)

Bose, S., Ozdenoren, E., Pape, A.: Optimal auctions with ambiguity. Theor. Econ. **1**, 411–438 (2006)

Bosmans, K., Ooghe, E.: A characterization of maximin. Econ. Theory Bull. **1**(2), 151–156 (2013)

Chakravorty, B., Corchón, L., Wilkie, S.: Credible implementation. Games Econ. Behav. **57**(1), 18–36 (2006)

Dasgupta, P.S., Hammond, P.J., Maskin, E.S.: The implementation of social choice rules: some general results on incentive compatibility. Rev. Econ. Stud. **46**(2), 185–216 (1979)

de Castro, L.I., Yannelis, N.C.: Ambiguity aversion solves the conflict between efficiency and incentive compatibility, working paper (2009)

de Castro, L.I., Yannelis, N.C.: An interpretation of Ellsbergs Paradox based on information and incompleteness. Econ. Theory Bull. **1**(2), 139–144 (2013)

de Castro, L.I., Pesce, M., Yannelis, N.C.: Core and equilibria under ambiguity. Econ. Theory **48**, 519–548 (2011)

de Castro, L.I., Liu, Z., Yannelis, N.C.: Implementation under ambiguity. Games Econ. Behav. (2015). doi:10.1016/j.geb.2015.10.010

de Castro, L.I., Liu, Z., Yannelis, N.C.: Relationship between partition and type model in general equilibrium with asymmetric information, working paper (2016)

de Castro, L.I., Pesce, M., Yannelis, N.C.: Rational expectations under Ambiguity, working paper (2015)

Gilboa, I., Schmeidler, D.: Maxmin expected utility with non-unique prior. J. Math. Econ. **18**(2), 141–153 (1989)

Glycopantis, D., Muir, A., Yannelis, N.C.: An extensive form interpretation of the private core. Econ. Theory **18**, 293–319 (2001)

Glycopantis, D., Muir, A., Yannelis, N.C.: On extensive form implementation of contracts in differential information economies. Econ. Theory **21**, 495–526 (2003)

Gollier, C.: Optimal insurance design of ambiguous risks. Econ. Theory **57**(3), 555–576 (2014)

Guo, H., Yannelis, N.C.: Ambiguous and robust full implementation, working paper (2015)

Hahn, G., Yannelis, N.C.: Coalitional Bayesian Nash implementation in differential information economies. Econ. Theory **18**, 485–509 (2001)

Hansen, L.P., Sargent, T.J.: Acknowledging misspecification in macroeconomic theory. Rev. Econ. Dyn. **4**, 519–535 (2001)

He, W., Yannelis, N.C.: Equilibrium theory under ambiguity. J. Math. Econ. **61**, 86–95 (2015)

Holmström, B., Myerson, R.B.: Efficient and durable decision rules with incomplete information. Econometrica **51**(6), 1799–1819 (1983)

[21] Each agent consumes his initial endowment, whenever their reports are not compatible. See, for example, Glycopantis et al. (2001), Glycopantis et al. (2003) and Liu (2016).

[22] That is, the mechanism designer has a private information set which consists of singletons.

Jackson, M.O.: Bayesian implementation. Econometrica **59**(2), 461–477 (1991)
Koufopoulos, K., Kozhan, R.: Optimal insurance under adverse selection and ambiguity aversion. Econ. Theory **62**(4), 659–687 (2016)
Liu, Z.: A note on the welfare of the maximin rational expectations. Econ. Theory Bull. **2**, 213–218 (2014)
Liu, Z.: Implementation of maximin rational expectations equilibrium. Econ. Theory **62**(4), 813–837 (2016)
Ohtaki, E., Ozaki, H.: Monetary equilibria and Knightian uncertainty. Econ. Theory **59**(3), 435–459 (2015)
Palfrey, T.R., Srivastava, S.: On Bayesian implementable allocations. Rev. Econ. Stud. **54**(2), 193–208 (1987)
Palfrey, T.R., Srivastava, S.: Implementation with incomplete information in exchange economies. Econometrica **57**(1), 115–134 (1989)
Traeger, C.P.: Why uncertainty matters: discounting under intertemporal risk aversion and ambiguity. Econ. Theory **56**(3), 627–664 (2014)
Wald, A.: Statistical Decision Functions. John Wiley and Sons, New York (1950)
Yannelis, N.C.: The Core of an economy with differential information. Econ. Theory **1**, 183–198 (1991)
Zimper, A., Ma, W.: Bayesian learning with multiple priors and nonvanishing ambiguity. Econ. Theory (2016). doi:10.1007/s00199-016-1007-y